

# Collections, bars and numbers: Analytical coincidence or Bach's design?<sup>1</sup>

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## The problem

The mathematical nature of Johann Sebastian Bach's works has entered popular myth. Claims about the supposed numerical bases of his compositions date back to the 1740s and are repeated in his obituary and subsequent biographies.<sup>2</sup> The problem is that we do not know what numbers or numerical processes these claims are referring to. As a result, they have attracted an array of wild and historically implausible explanations about Bach's use of numbers. In 1947, Friedrich Smend contributed his notorious number alphabet theory to the debate. Typical of his writing is:

CHRISTUS is expressed by the number 112, CREDO by 43. The 'Credo' of the B Minor Mass, in its original form, has 784 bars (i.e.  $7 \times 112$ ) . . . In the chorus 'Credo in unum Deum' in the same work, the word CREDO appears 43 times. The same chorus, plus the following movement 'Patrem omnipotentem' amounts to a total of 129 bars ( $3 \times 43$ ).<sup>3</sup>

In 1991, I showed that Smend's colourful and extreme interpretations could be disregarded, as they were based on his misunderstanding of the nature of the number alphabet in Bach's time.<sup>4</sup> There is no doubt that this aspect of Smend's work is deeply flawed, and yet because of one small feature in his theory, I could not reject the whole. Smend worked on the presupposition that Bach had planned the length of his movements and works. In 1947 this presupposition was not only unproven, it was wholly unexplored. It has been accepted for many years that the

<sup>1</sup> Issues explored in this paper will be covered in detail in my forthcoming book *Bach's Numbers Explained* (working title).

<sup>2</sup> *The New Bach Reader*, ed. David and Mendel, rev. Wolff (New York: W W Norton, 1998), p. 297 (Mizler and Mattheson) and pp. 417-82 (Forkel). This source is hereafter abbreviated as *NBR*

<sup>3</sup> Friedrich Smend, *Johann Sebastian Bach: Kirchenkantaten erläutert*, 6 vols. (Berlin: Christlicher Zeitschriftenverlag, 1947-9; reprint edns Berlin, 1950 and 1966), III, p. 20.

<sup>4</sup> Ruth Tatlow, *Bach and the Riddle of the Number Alphabet* (Cambridge: Cambridge University Press, 1991, paperback reprint 2006).

number of bars in a composition is a simple by-product of the working out of musical ideas, i.e. the composer ends the composition when the working out of musical ideas is complete. However, if Smend's presuppositions were correct and Bach had exercised strict control over the number of bars in his compositions, then the manuscript scores would yield data hitherto ignored by musicologists. But rather than trying to suggest interpretations, as Smend did, it was necessary first to establish whether a composer, and specifically Bach, actually did exercise conscious control over the number of bars in a work. And only if this were the case, would it be appropriate to ask whether the number alphabet, or a different bar-based technique, had any extra-musical meaning, and might be the numerical basis that Bach's contemporaries claimed lay at the heart of his compositions.

The prospect of devoting research time to trying to answer such elusive questions seemed little better than academic suicide. There was a high probability of generating random numbers and drawing no serious conclusion; the roll-call of self-deluded number hunters over the past millennium was a clear enough warning. Nevertheless, the slim possibility that the investigation might result in a new theory of Bach's compositional procedure which could explain hitherto mystifying statements swung the balance. I decided to proceed cautiously in the knowledge that at any point I could call it a day and close the books.

## I Conscious planning?

### i) Descriptions of compositional planning

Music treatises written or published in Bach's time are the most obvious place to search for a description of how Bach and his contemporaries composed. One of the most prolific music theorists of this period was Johann Mattheson, whose clearest descriptions of compositional planning appear in *Der vollkommene Capellmeister*.<sup>5</sup> Although Bach was based in Leipzig and Mattheson in Hamburg, they knew each other's work. Their creative relationship was fostered not only by frequent indirect exchanges via Telemann, but by the availability of their publications at the Hamburg and Leipzig fairs.<sup>6</sup> The fact that they were acquainted does not prove that they thought alike, but in the absence of any statement by Bach on compositional planning, Mattheson's formulations are important as they were based on contemporary compositional practices.

Mattheson's most detailed description of compositional planning extends over three pages. His first recommendation to the Capellmeister is to sketch a composition, just as an architect draws up plans for a building.

. . . order all the parts and details of a melody or of an entire melodic composition, just as you would design a house, making a sketch or

<sup>5</sup> Johann Mattheson, *Der vollkommene Capellmeister* (Hamburg, 1739)

<sup>6</sup> George B. Stauffer, 'Johann Mattheson and J. S. Bach: the Hamburg Connection', *New Mattheson Studies*, eds. George J. Buelow and Hans J. Marx (Cambridge: Cambridge University Press, 1983), pp. 353–68.

plan to show where a room, a parlour, a chamber etc. should be placed.<sup>7</sup>

Mattheson tells the composer to aim for similar proportions in all the parts, not only so that the composition is pleasing to the senses, but also so that it is durable.<sup>8</sup> He describes how this can be done, although he is aware that it takes time:

Whoever wishes to use the method described above should outline his complete project on a sheet, sketch it roughly and arrange it in an orderly manner before he proceeds to the elaboration. In my humble opinion this is the best way of all to ensure that each part will demonstrate a specific proportion (*Verhältniß*), uniformity and agreement: for nothing in the world is more pleasing to the ear.<sup>9</sup>

Musical proportions, demonstrating that mathematics lies at the basis of all music, are the starting point of the vast majority of music treatises published before the mid-eighteenth century.<sup>10</sup> The proportions usually relate to musical pitch and intervals, and were also used to describe rhythm. In the passages quoted above, Mattheson is describing a different use of proportions: to order all the parts of a musical composition as an architect would the parts of a house.<sup>11</sup> In Bach's day architectural plans were detailed, with images of the building in plan and section illustrated by specific units of measurement. To draw up a well-proportioned groundplan, as Mattheson recommends, the composer would also need specific measurements. Certain measurements were obvious. For example, the composer could decide to have six movements in a work, or six works in a collection, as an architect could decide on six rooms in a building or six buildings in a complex. But the architect would then decide the exact dimensions of the room or building, to make sure that it was well-proportioned. How could the composer do the equivalent and organise the exact dimensions within his piece of music, so that 'each part may demonstrate a specific proportion' ('eine gewisse Verhältniß')?<sup>12</sup> This very clear direction would require specific measurements. The word *Verhältniß* was a mathematical term, as the eight columns of definitions in

<sup>7</sup> Mattheson, *Capellmeister* (1739), p. 234 §4.

<sup>8</sup> *ibid.* p. 240, § 29.

<sup>9</sup> *ibid.*, p. 240 §30 'Wer sich also, seiner Fertigkeit im Setzen ungeachtet, der oberwehnten Methode, auf gewisse ungezwungene Art bedienen will, der entwerffe etwa auf einem Bogen sein völliges Vorhaben, reisse es auf das gröbste ab, und richte es ordentlich ein, ehe und bevor er zur Ausarbeitung schreitet. Meines wenigen Erachtens ist diese die allerbeste Weise, dadurch ein Werck sein rechtes Geschicke bekömmt, und ieder Theil so abgemessen werden kan, daß er mit dem andern **eine gewisse Verhältniß**, Gleichförmigkeit und Uebereinstimmung darlege: maassen dem Gehör nichts auf der Welt lieber ist, denn das. §31 Zeit und Gedult wollen dazu gehören.' In *Musurgia Universalis* (Rome, 1650), p. 193f. Athanasius Kircher had described a similar method of composing, which he termed 'Plectrologia Musarum'.

<sup>10</sup> The focus on proportions owes much to Boethius (480–524), who was quoting the work of mathematician Nicomachus (c.100), and also to Pythagoras.

<sup>11</sup> The architectural metaphor is repeated by other theorists of the period, including Bach's fellow Mizler society member, Meinrad Spiess.

<sup>12</sup> Mattheson, *Capellmeister* (1739), p. 240 §30.

Zedler's dictionary clearly show.<sup>13</sup> When Mattheson used the word *Verhältniß* he meant a mathematical proportion, and his readers understood this.<sup>14</sup>

Forkel also used the word *Verhältniß* in his famous biography, when he described Bach's method of composing. Since Forkel claims that much of his source material came directly from Bach's two elder sons, we have to take his anecdotes seriously. Among the patriotic sentiment and over-generous admiration for Bach, Forkel describes how the young Bach refined his compositional technique:

He soon began to feel that the eternal rushing and leaping led to nothing; that there must be order, connection and proportion ('das Ordnung, Zusammenhang und Verhältniß') in the thoughts, and that, to attain such objects, some kind of guide was necessary. Vivaldi's Concertos for the violin . . . served him for such a guide. He so often heard them praised as admirable compositions that he conceived the happy idea of arranging them all for his clavier. He studied the chain of ideas, their relation to each other ('das Verhältniß derselben unter einander'), the variations of the modulations, and many other particulars.<sup>15</sup>

Bach made these transcriptions (BWV 972-987) sometime between 1713 and 1714. Is it possible that he used them as a study in forming proportions between a collection?<sup>16</sup> Did Forkel and Mattheson understand the word *Verhältniß* to convey a specific unit of measurement? And if so, which unit of measurement did they have in mind? Was it the bar, or was it a unit of time or some other unit? Michael Praetorius and Lorenz Mizler shed some light on this question.

## ii) The bar and the minute

In Bach's time in Leipzig, the duration of the music in the main Sunday morning service was a matter of concern to both clergy and musicians. It was important that the music and liturgy at the beginning of the service were timed so that the sermon could begin at the stroke of 8 o'clock.<sup>17</sup> Church musicians had faced similar restrictions for more than a century. In 1619, Michael Praetorius proposed a practical solution: a method of calculating the duration of a composition in units related to fifteen minutes. He reckoned that 80 tempora of average metre

<sup>13</sup> Johann Heinrich Zedler, *Großes vollständiges Universal Lexicon aller Wissenschaften und Künste*

(Leipzig, 1732-54), online <http://www.zedler-lexikon.de>. Volume 47, columns 790-98.

<sup>14</sup> Modern translations frequently use the more general term 'relation' or 'relationship', which masks the specifically mathematical meaning.

<sup>15</sup> *NBR*, p. 441-2. N.B. The modern translation uses the word 'relation' for *Verhältniß*. This is misleading and contradicts the mathematical definitions given in Zedler (see footnote 11).

<sup>16</sup> See numerical result of the concerto transcriptions on p. 20 below.

<sup>17</sup> Günther Stiller, *Johann Sebastian Bach and Liturgical Life in Leipzig*, transl. from the original German by Bouman, Poellot and Oswald (St Louis, Concordia Publishing House, 1970) p. 124, quoting *Leipziger Kirchen-Staat: Das ist, Deutlicher Unterricht vom Gottes-Dienst in Leipzig* (Leipzig, 1710), p. 7: 'The normal time for the Sermon is one hour, and the priest usually closes the Sermon at nine o'clock or shortly after'.

would last 7½ minutes, 160 bars 15 minutes, 320 bars 30 minutes, and so on.<sup>18</sup> In 1754, Lorenz Mizler published similar, albeit updated, guidelines in an article describing Bach's contribution to the Society of Musical Science.<sup>19</sup> The statement on compositional duration appears directly after the announcement of Bach's presentation of the Canon 'Triplex a 6 voci' (BWV 1076) to the Society, giving the impression that the statement too may have been contributed by Bach:

. . . From experience one can determine the length of a cantata, so that 350 bars, of any metre, will last approximately 25 minutes, which is long enough in winter, although in summer it could be 8–10 minutes longer and be roughly 400 bars long. But it's my opinion that a composer should think more about the music or movement, bringing it into beautiful order, rather than about the time. It should not depend on specific minutes.<sup>20</sup>

In Mizler's example, the equation is probably 14 bars per minute. There is in fact a problem with the arithmetic, possibly caused by a misprint. If 350 bars last 25 minutes, then 490 and not 400 bars last 35 minutes.<sup>21</sup> Be that as it may, this equation can be used only as a very rough guide to the duration of a composition, not simply because changes in time signature and speed are ignored, but because Mizler encourages the composer to attach more value to beautiful order than to specific timing.

Praetorius and Mizler provide us with documentary evidence that the bar (or tempora) was used as a unit of measurement to estimate the approximate duration of a cantata. The number of bars in a work does not change, whereas the duration of a work changes with every performance. Is it not probable, therefore,

<sup>18</sup> *Syntagma Musicum* vol. III, *Termini Musici* (Wolffenbüttel, 1619), Facsimile reprint edition, *Documenta Musicologica* ed. Wilibald Gurlitt (Kassel, Basel, 1958), pp. 87–8. Allhier wil ich auch dieses erinnern: Dass ich in den General Bassen allezeit am ende eines jeden verzeichnet hat wie viel tempora ein jeder Gesand auch ein jeder Theil oder part Cantionis in sich halte. Denn weil ich nothwendig observieren muessen wie viel tempora, wenn man einen rechten mittelmässigen Tact helt in einer viertel Stunde musiciret werden koennen: Als nemblich 80 tempora in einer halben viertel Stunde, 160 tempora in einer ganzen viertel Stunde, 320 tempora in einer halben Stunde, 640 tempora in einer ganzen Stunde. So kan man sich desto besser darnach richten wie lang derselbe Gesang oder Concert sich erstecken moechte darmit die Predigt nicht remorirt sondern zu rechten zeit angefangen auch die andere Kirchen Ceremonien darneben verrichtet werden koennen.

<sup>19</sup> Mizler's society for intellectually-minded musicians was founded in 1738. It was not open to all, but only to those invited by Mizler to become members. Members were sent discussion-material such as musical news, essays, compositions and theoretical writings by postal package. Bach joined the society in 1747 and his first compositional contribution was the triple canon for six voices BWV 1076. See Christoph Wolff, *Johann Sebastian Bach: the Learned Musician* (Oxford: Oxford University Press, 2000), p. 422.

<sup>20</sup> Lorenz Mizler, *Musikalischer Bibliothek* (Leipzig, 1754), Vol. 4 Part 1, p. 108: 'Im fünften Packet der Societät hat der seel. Capellm. Bach eine dreyfache Kreisfuge mit sechs Stimmen zur Auflösung vorgeleget. Siehe Tab. IV. fig. 16. Auch sind die Texte zu den Kirchencantaten beurtheilt, und dabey verschiedenes nütliches erinnert worden. Wir wollen den Kern davon hier beybringen, zum Nutzen der Kirchencomponisten, und der Poeten, so für die Musik geistliche Gedichte machen. Im Winter sollen die Kirchenmusiken etwas kürtzer seyn als im Sommer etc.'

<sup>21</sup> It is not difficult to see how a badly written 9 could be misread as a 0 (490 as 400). Unfortunately vol. 4 has no page of corrections.

that a composer seeking to create beautiful structural proportions in a composition would choose the unchanging medium above the changing? That is, that he would create proportions through the number of bars rather than the duration of the piece? We are not told. The theorists are silent on this point. But they do tell us that there was at least one practical reason for knowing the total number of bars in a work.

### iii) The copyist

In order to make an accurate copy of a score, the composer or copyist had to be aware of the number of bars in a movement. The number of bars would help the copyist both to make an economic disposition of the bars on a page, and to ensure that the original and the copy were identical. In scores and parts of this period there are many examples of the copyist recording the bar count at the bottom of a page, or at the end of a movement. Copying music was an everyday necessity, perhaps even drudgery, common to pupil, apprentice and master. Paper was expensive. Printed manuscript paper was not yet common, and the art of economising on the space used was encouraged. We know that Bach owned several different widths of rastrum, which he chose carefully when ruling the staves of his score. We know too, from figures written in the original scores, that Bach and his copyists frequently counted the number of bars in movements. For example, bar numbers appear in three movements in the autograph score of the *B-Minor Mass*, P 180. In the 'Quoniam', Bach or a copyist wrote the figure 94 by the last bar of page 72, which is bar 94, and at the end of the first line of page 73 the figure 100, which is bar 100. In the 'Confiteor' he or a copyist wrote the figures 61, 100 and 141 at the end of each of pages 137, 138 and 139, corresponding to bars 61, 100 and 141, and on page 105 the copyist Michel wrote the number 84 at the end of the 'Credo', which has 84 bars.<sup>22</sup> Furthermore, the handwriting in the 'Quoniam' shows that Bach was either composing this movement directly into the score or partially copying it from a pre-existing source.<sup>23</sup> Since we also know from the Dresden parts that Bach made the copies for this movement, it shows that the numbers were either a composing aid or a copying aid; that is, if it was he who wrote the bar numbers on pages 72 and 73. Even though we cannot know exactly why the numbers are written in the score, they are nonetheless important as they show that Bach and his copyists were conscious of the cumulative total of bars in a movement. It thus follows that Bach or his copyists, if they wished, could follow the guidelines by Praetorius and/or Mizler to calculate the approximate duration of the work. It also shows that Bach could have used the figures to create pleasing order and beautiful proportions, either while he compiled the work or when he later revised it. But did he?

<sup>22</sup> In *Kirchenkantaten* (Berlin, 1947, repr. 1966) IV, p. 14, Smend offered an elaborate interpretation of this handwritten number 84, assuming in good faith that Bach had written it. A check against C. P. E. Bach's parts St 118, written after J. S. Bach's death, shows that the same hand wrote the figure 84 in the corresponding place in the soprano solo part, St 118/ 2, thus ruling out the possibility that J. S. Bach wrote it. I am grateful to Yo Tomita for many stimulating and helpful discussions on this subject.

<sup>23</sup> George B. Stauffer, *Bach the Mass in B minor* (Yale: Yale University Press, 2003), pp. 134-5.

#### iv) Observation and experimentation

One of my first experiments was based on the numerical structure of the *St Matthew Passion*. I chose this work as there are surviving manuscripts of both an early and late version.<sup>24</sup> Mattheson's injunction to sketch a work before composing suggests that any conscious numerical structure would be formed pre-compositionally. Since Mattheson also recommended the composer to study the poetic text of a work before beginning to compose, it seemed not unreasonable to test the idea that Bach began by setting the poetic texts from the libretto, for which he composed arias, before he set the fixed biblical verses to which he set recitatives. I was looking for any evidence of numerical ordering within these arias. My sources were what was then considered to be a copy of the early version in Altnickol's hand, but which we now know to be in Farlau's hand,<sup>25</sup> and Bach's revised score, P25.

The experiment threw up two particularly intriguing results. The first was that the total number of bars in the arias for tenor, bass and choir form a perfectly balanced unity across the work, with 410 bars in Part One and 410 bars in Part Two. It is a result that can be seen only in the early version, where the aria 'Ach nun ist mein Jesus hin' was scored for bass soloist. When Bach revised the score, he transcribed the aria for alto, therefore destroying the original totals. Was this perfect 1:1 proportion evidence of Bach's compositional procedure, or a fluke of the arithmetic? And if Bach destroyed the proportion by changing the soloist, what status, if any, did the proportion have?

<b>Part One</b>		<b>Bars</b>	
1 Choir	Kommt ihr Töchter	90	
20 Tenor + Choir.	Ich will bey meinem Jesu wachen	81 (DC10)	
23 Bass	Gerne will ich mich	102 (DC72)	
27 Sop.Alt + Choir	So ist mein Jesus/Sind Blitze	137	<b>410 bars</b>
<b>Part Two</b>			
30 Bass +Choir	Ach nun ist mein Jesus hin!	123	<b>b</b>
35 Tenor	Geduld!	47	<b>a</b>
42 Bass	Gebt mir meinen Jesum wieder	53 (DC 12)	
57 Bass	Komm süßes Kreuz	54	
65 Bass	Mache dich, mein Herze rein	53 (DC 28)	
68 Choir	Wir setzen uns mit Thränen nieder	80 (DC 48)	<b>c 410 bars</b>

Example 1: Proportion of 1:1 in Parts One and Two of *St Matthew Passion*

A second result showed that all the free-texted arias from Part Two form a 1:1 proportion, with 330:330 consecutive bars.

<sup>24</sup> Ruth Tatlow, 'Towards a Theory of Bach's Pre-Compositional Style', *Bach und die Stile: Bericht über das 2. Dortmunder Bach-Symposium 1998*, ed. Martin Geck (Dortmund, Klangfarben Musikverlag, 1999), 19–36.

<sup>25</sup> Andreas Glöckner, 'Zur Wiederentdeckung der Matthäus-Passion im Jahre 1829', *Bach-Jahrbuch* 90 (2004), 133–55.

<b>Part Two</b>		<b>Bars</b>
30 Bass	Ach nun ist mein Jesus hin!	123
35 Tenor	Geduld!	47
39 Alto	Erbarme dich	46 (DC 8)
42 Bass	Gebt mir meinen Jesum wieder	53 (DC 12)
49 Sopr.	Aus Liebe will mein Heyland sterben	61 (DC 13) <sup>26</sup> <b>330 bars</b>
52 Alto	Können Thränen meiner Wangen	91 (DC 64)
57 Bass	Komm, süßes Kreuz	54
60 Alto	Sehet, Jesus hat die Hand	52 (51 considered to be error)
65 Bass	Mache dich, mein Herze, rein	53 (DC 28)
68 Chorus	Wir setzen uns mit Thränen nieder	80 (DC 48) <b>330 bars</b>

Example 2: Proportion of 1:1 within Part Two of *St Matthew Passion*

There is no doubt that these figures are beautiful, but are they evidence of Bach's composing method? The results raised more questions than they answered. Even though the experiment was restricted, there were too many variables in the scores and between versions to produce reliable data or to be able to trust the results. For example, it was not clear whether the 51-bar version of 'Sehet, Jesus hat die Hand' in Farlau's score was a copying error or Bach's original intention; if the former, then the figures become 330:329. At this stage, I was also unsure whether there was any validity in counting the arias without the *da capo* return. The composer as copyist knew how many bars of score-space each movement required, i.e. he knew the length of each movement without its repeated *da capo* section, but I was reluctant to accept that Bach would have used such a pragmatic unit which cut across the musical meaning.

A later observation of 329 and 330 bars in the structure of the 'Missa' of the *B-Minor Mass* was intriguing, as it was so close to the observation in the *St Matthew Passion*, but the results were complicated by the fact that two movements were in the ambiguous *stile antico* notation. In the score of the 'Gratias', for example, bar lines form a 46-bar movement, whereas in Bach's Dresden parts, this movement is clearly barred in semibreves, making it 92 bars long.

1 Kyrie	C	126	
2 Christe	C	85	
3 Kyrie	C slash (notated 59 bars)	118	<b>329 bars</b>
4 Gloria	3/8 Et in terra pax C	176	
5 Laudamus te	C	62	
6 Gratias	C slash (notated 46 bars)	92	<b>330 bars</b>

Example 3: Proportion almost 1:1 in six consecutive movements of *B-Minor Mass*

<sup>26</sup> AmB 6,7 is written as 61 DC 13, whereas P25 as 62 DC 12.



Similar experiments and observations in other works served only to increase the growing list of queries. I constantly had to question my numerical observations. Was I looking at Bach's compositional design, or were these numbers a result of my brain's desire for order and pattern? The history and interpretation of the autograph scores were complex, and the many numerical permutations in the large scores unwieldy. The methodology was too experimental, and I decided to put the results to one side.

At about this time I discovered that the *St Matthew Passion* (P25) and the *Brandenburg Concertos* (Am.B.78) have exactly 2800 and 2500 bars respectively. The utter simplicity and precision of the figures was astounding. I found these figures in the recently published Schmieder's *Kleine Ausgabe*,<sup>27</sup> where the bar numbers are given without repeats and yet include all the bars in both the ritornello and full *da capo* returns. What, if anything, did these figures tell us about Bach's compositional procedure, and how did they relate to my experimental results? Was Bach aware of these totals, or were they a freak of the Neue Bach Ausgabe editorial policy?

Then there was the vexed issue of the so-called 'golden section'. Was Bach interested in this proportion? It has a noble reputation for beauty, but would Bach have used it? This was a necessary but time-consuming digression, and the results were convincing.<sup>28</sup> Although Bach knew Euclid's ratio, and the additive sequence later named after Fibonacci, there is no evidence whatsoever to suggest that Bach or any other artists of the time had any interest in its aesthetic properties. This conclusion is confirmed by the almost dismissive brevity of the entry 'sectio divina' in Zedler's *Lexicon*, which describes Euclid's ratio in a two-sentence geometrical definition mentioning neither numerical expression nor aesthetic application.<sup>29</sup> This conclusion was confirmed by the lack of any such entry in J. G. Walther's musical dictionary.<sup>30</sup>

Walther did, however, include an entry for a mathematical term which he defined before referring the reader to a musical treatise and to Euclid's theorem. The entry is recorded under 'numerus perfectus'.<sup>31</sup> Why musicians of this period should be interested in perfect numbers and their properties, and what their precise musical application was, are questions still to be answered.<sup>32</sup> However, it is an interesting negative proof, since Walther omitted such an entry for Euclid's 'sectio divina'.

<sup>27</sup> *Bach-Werke-Verzeichnis: Kleine Ausgabe*, nach der von Wolfgang Schmieder vorgelegten 2. Ausgabe, ed. Alfred Dürr, Yoshitake Kobayashi, Kirsten Beißwenger (Wiesbaden: Breitkopf & Härtel, 1998).

<sup>28</sup> 'The Use and Abuse of Fibonacci Numbers and the Golden Section in Musicology Today', *Understanding Bach*, 1 (2006), 69–85, online <http://www.bachnetwork.co.uk/ub1/tatlow.pdf>

<sup>29</sup> Johann Heinrich Zedler, *Großes vollständiges Universal Lexicon aller Wissenschaften und Künste* (Leipzig, 1732–54), online <http://www.zedler-lexikon.de>, s.v. 'section divina'. A massive work covering 68,000 pages in 68 folio volumes.

<sup>30</sup> Johann Gottfried Walther, *Musicalisches Lexicon oder Musicalische Bibliothec* (Leipzig, 1732).

<sup>31</sup> A *numerus perfectus* or a perfect number is one whose divisors add up exactly to the number itself. They are extremely rare: the first four perfect numbers are 6, 28, 496 and 8128. See also p. 54 below.

<sup>32</sup> The publicity given to the golden section has cast a smoke screen over this small, but potentially significant entry.

### v) Consistency of counting

It was clear from my experimentation and observation of numbers in Bach's scores that, if there was to be any hope of continuing the investigation, an objective control must be set up. Without methodological consistency, it was impossible for me to establish whether the results were numerical coincidence, evidence of Bach's working method, or even my own unconscious manipulation of the figures.

Setting up an objective control necessitated discovering what exactly a 'bar', or 'takt' meant to Bach, Mizler and Mattheson, and whether a composer would count the bars that were written on the page, or those that were performed. As we have seen, the copyist needed to know how many bars would fit on to the page in order to plan a convenient disposition of the score. A movement with a *da capo* indication obviously has two different bar totals: the number of bars written and the number of bars performed. The total number of bars in a work with several *da capo* arias therefore changes drastically according to the method of counting. Furthermore, Bach himself was frequently inconsistent in his notation of a *da capo* or *dal segno* section, sometimes writing the repeated section in full, and sometimes using a *da capo/dal segno* indication.<sup>33</sup> Observing the repeats in a work also gives a much larger bar total, which in a multi-movement work seems unwieldy and unattractive to me, but would it have been unattractive to Bach? There is also the question of how to define the bar when there are two lengths of bar-line in a movement. For example, in movements notated in *stile antico* C slash, Bach frequently added a full-length bar-line every breve, and a shorter vertical line after each semibreve. He occasionally notated a fast 3/8 movement in this way too, with a short bar-line every three quavers and a full-length line after six.<sup>34</sup> And of course there was the variable of Bach himself. Was it realistic to expect his method of counting, ordering and creating proportions to remain consistent throughout his composing life, if indeed he used such a method? All these questions needed to be asked against the background of the major query: **if** Bach had been ordering the work through the number of bars. I did not yet know.

I had promised myself that at any point I could call it a day and close the books. That point had arrived. Or so I thought, but my forbearance was to be tested a little further. I received an invitation to write a paper at short notice on the numerical structure of the *Six solos* for violin (BWV 1001-1006). This commission proved to be the catalyst that would yield results to allow not only the development of a consistent method of counting, but the development of some firm principles against which to test all Bach's works.<sup>35</sup> The final result was the formulation of proportional parallelism.

<sup>33</sup> This inconsistency is more frequent in short ritornello sections. It is rare for Bach to write out in full a complete A section in a *da capo* ABA aria.

<sup>34</sup> BWV 1001/4 Presto.

<sup>35</sup> I am extremely grateful to Dominik Sackmann for commissioning the article, and to the Hinrichsen Foundation for sponsoring the research and writing, without which I would never have made these findings and discovered proportional parallelism.

## II Formulation of a theory

The *Six solos* demonstrate a remarkable series of proportions that exists concurrently at different constructional levels, numbered below as levels 1-5. The results are based on data from the autograph score, P967, signed and dated 'Joh: Seb: Bach ao.1720'. I have also consulted the copies by Anna Magdalena Bach, P268 and J. P. Kellner, P804.

### i) Proportion level 2: formed between movements of a work

A clear 1:1 and 1:2 proportion can be seen in the bar structure of the first solo, the sonata in g minor (BWV 1001). On the written score, the first three movements (Adagio, Fuga and Siciliano) have 136 bars, while the fourth movement (Presto) has 136 bars, thus creating a 1:1 proportion (see Example 4).

		<b>Bars (with repeats)</b>		<b>Bars (no repeats)</b>	
<b>Sonata 1</b>	Adagio	22		22	
	Fuga	94		94	
	Siciliana	20	<b>136</b>	20	<b>136</b>
	Presto	272	<b>272 1:2</b>	136	<b>136 1:1</b>

Example 4: Proportion 1:1 and 1:2 in sonata in g minor (BWV 1001)

In performance, due to repeats in the Presto the violinist plays 136 bars in the first three movements and 272 bars in the fourth, thus creating a 1:2 proportion.<sup>36</sup>

Something very similar happens in the second solo, the partita in b minor (BWV 1002). The partita has eight movements, formed by four movements and their respective doubles. On the written score the first six movements have 272 bars, while the final two movements have 136 bars, thus creating a 2:1 proportion (see Example 5). There are repeat indications in every movement, and so when these are observed the violinist still creates a 2:1 proportion, playing 544 bars in the first six movements and 272 bars in the fourth.

		<b>Bars (with repeats)</b>		<b>Bars (no repeats)</b>	
<b>Partita 1</b>	Allemande	48		24	
	Double	48		24	
	Corrente	160		80	
	Double	160		80	
	Sarabande	64		32	
	Double	64	<b>544</b>	32	<b>272</b>
	T di Borea	136		68	
	Double	136	<b>272 2:1</b>	68	<b>136 2:1</b>

Example 5: Proportion 2:1 within partita in b minor (BWV 1002)

<sup>36</sup> This performed proportion would not be detected in time, as the movements are at different speeds. The proportion appears on paper only, even in the so-called 'performed' proportions.

It is interesting that this second solo is missing in Kellner's copy of the solos, P804, which may indicate that it was the last to be composed or added to the collection.

The sixth solo, the partita in E major, is the only other work in this collection to have a level 2 proportion. The Preludio, Loure and either Gigue or Menuet 2 have 194 bars, and the remaining four movements have 194 bars (see Example 6). This result on its own is not spectacular, and could easily be dismissed as arithmetical coincidence, but because this type of proportion, formed with few terms, is seen repeatedly across Bach's works, it must be taken seriously. The proportion may also hold a clue to the compositional history of the collection.

	<b>Bars (no repeats)</b>		
<b>Partita 3</b>	Preludio	138	
	Loure	24	
	Gavotte	92	
	Menuet 1	34	
	Menuet 2	32	
	Bouree	36	
	Gigue	32	
		<b>194</b>	<b>194 1:1</b>

Example 6: Proportion 1:1 within partita in E major (BWV 1006)

The omission of the final bar of the E major Preludio (BWV 1006/1) is also a possible clue to the collection's compositional history and evidence of Bach manipulating the numerical total to achieve the proportion. It is not unusual for these changes to occur in the sixth or final work in a collection.<sup>37</sup> The supposedly 'early' version of the Preludio, preserved in Bach's autograph copy of the lute suite (BWV 1006a), has 139 bars, with the final arpeggiated flourish concluding logically with a dotted minim on the first beat of bar 139 (see Example 7).



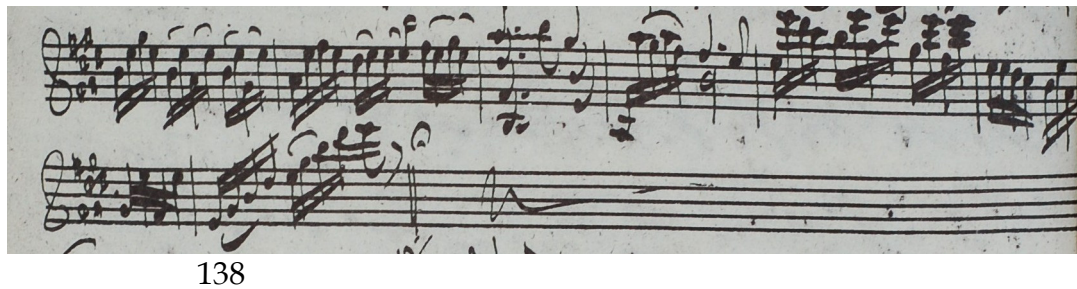
Example 7: BWV 1006a/1 <sup>38</sup>

When seen against the regularity of the final phrase of the lute version, one notices the irregularity of the violin version with its implied 'missing' final bar

<sup>37</sup> Hans Eppstein, 'Fragen der Ordnungsprinzipien in Bach's Köthener und Leipziger Instrumentalsammlungen', *Leipziger Beiträge zur Bach-Forschung: Leipzig 2000*. ed. Ulrich Leisinger (Hildesheim: Olm, 2002), 132-5. See also note 58 below.

<sup>38</sup> Reproduced from a facsimile of the autograph held in Musashino College, Tokyo.

(see Example 8). Is this evidence of Bach's manipulation to achieve his proportional plan?



Example 8: BWV 1006/1<sup>39</sup>

### ii) Proportion level 3: formed between two works in a collection

A 1:2 and 2:3 proportion is formed between the first and second works in the collection. When performed with all the repeats, the violinist plays 418 bars in the g minor sonata and 816 bars in the b minor partita. On the score a 2:3 proportion is formed by 272 and 408 bars (see Example 9).

	<b>Bars (with repeats)</b>	<b>Bars (no repeats)</b>
G minor sonata (BWV 1001)	408	272
B minor partita (BWV 1002)	816	408

Example 9: Proportion level 3: 1:2 and 2:3 within the first sonata and partita

### iii) Proportion level 4: formed in the collection as a whole

The whole collection also forms a perfect 2:1 proportion. The *Six solos* cover 2400 written bars: four solos have exactly 1600 bars and the remaining two solos have 800 bars. The 2:1 proportion exists not only in the bar total 1600:800 but also in the number of works 4:2 (see Example 10).

	<b>Bars (no repeats)</b>	
S1 (Sonata in g minor)	272	
P1 (Partita in b minor)	<b>b</b> 408	
S2 (Sonata in a minor)	<b>a</b> 396	
S3 (Sonata in C major)	<b>c</b> 524	<b>1600 bars</b>
P2 (Partita in d minor)	412	
P3 (Partita in E major)	388	<b>800 bars</b>

Example 10: Proportion level 4: 2:1 across the *Six solos* (BWV 1001-1006)

From the evidence we have today, it is not possible to ascertain when Bach decided to create this 2:1 proportion within the whole collection. If Kellner's copy

<sup>39</sup> There are many facsimile reproductions of this work.

is an earlier version, then we have certain fixed points. In Kellner's copy there is no b minor partita, several movements are missing, and both the Fuga in the g minor sonata and the Ciaconna in the d minor partita are abbreviated, although the E major Preludio has 138 and not 139 bars. However, there are questions about the status of Kellner's copy. Bach's signed and dated autograph, P967, on the other hand, contains important clues as to how he might have constructed such a large-scale work.

The d minor Ciaconna appears as the fifth movement in an otherwise perfectly ordinary partita of four movements. Appearing after the Giga, which is traditionally the final movement of a partita, the Ciaconna stands out not simply because of its position, but because of its length. Its 257 bars overwhelm the entire structure of the partita, as the remaining four movements together have just 155 bars. The same imbalance exists in the second and third sonatas, where we find massive fugues overpowering the otherwise simple structure. The a minor Fuga has all of 289 bars, with its sister movements having just 105 bars: the Fuga is more than twice the length of the remaining work. Similarly, the 354 bars of the C major fugue dominates the third sonata, which otherwise has just 170 bars. Violinist and audience alike notice these three gigantic movements when they are played in the context of either the individual solo or the whole collection. Intriguingly, these mammoth movements have exactly 900 bars (see Example 11).

	<b>Bars (no repeats)</b>
Fuga (Sonata in a minor)	289
Ciaconna (Partita in d minor)	257
Fuga (Sonata in C major)	354 <b>900 bars</b>

#### Example 11: 900 bars of construction

In Bach's works and collections, large-scale units which form a whole round number are not uncommon at this level of construction. Are such large-scale units evidence of Bach setting himself a compositional constraint in order to achieve his desired proportion? If so, there are implications for the compositional history of the collection. The 900 bars unit was formed after he had composed the abbreviated version of the Ciaconna which appears in Kellner's copy. Did Bach construct these together? Or is this little piece of numerical evidence sufficient to suggest that Kellner's copy was simply corrupt, rather than a faithful copy of an early version?

#### iv) Proportion level 5: formed between two collections

There is one more level of proportion, which is perhaps the most bizarre of all the proportions present in Bach's collections. Several of Bach's revised collections have exactly the same, or exactly half, the number of bars as another collection, which I term a level five proportion. For example, the 3120 bars of his first two perfectly copied keyboard collections, *Das Wohltemperirte Klavier* (BWV 846-869) and *Aufrichtige Anleitung* (BWV 772-801) form a perfect 1:1 proportion with the 3120 bars of his first published collection, *Clavierübung I*, which he considered his

Opus 1, and his second, *Clavierübung II*. The proportion is formed between two discrete keyboard collections surviving in fair copy and two discrete keyboard collections in published form. They are united by an identical bar total, and in this sense are proportionally parallel.

In the case of the *Six solos* for violin a level five 1:1 proportion is formed with the *Six sonatas* for harpsichord and violin (BWV 1014-1019), which has exactly 2400 performed bars.<sup>40</sup> The *Six sonatas* also have a level four proportion identical to that in the *Six solos*: the level four 2:1 proportion in the *Six sonatas* is formed between the number of sonatas (4:2) as well as between the number of bars in the collection 1600:800 (see Example 12). This is a particularly interesting case, as three different versions of the final sonata (BWV 1019) have survived. The version that displays the proportions is the copy by Altnickol, P229, considered to be the latest. The numerical results seem to confirm the conclusion drawn through source studies, that Altnickol's version represents Bach's latest revision. The results also show that Altnickol's version is faithful to Bach's lost finally-revised

		<b>Bars (with repeats)</b>
S1 (Sonata in b minor)	b	328
S2 (Sonata in A major)	a	419
S4 (Sonata in c minor)	c	477
S6 (Sonata in G major)		376
		<b>1600 bars</b>
S3 (Sonata in E major)		397
S5 (Sonata in f minor)		403
		<b>800 bars</b>

Example 12: Proportion level 4: 2:1 across the *Six sonatas* (BWV 1014-1019)

original score, and demonstrates one way in which the new theory can provide the editor with an additional technique to check three possible versions of a score.

Many scholars have surmised that Bach intended there to be a Libro Secondo to his Libro Primo, the *Six solos*. It is usually thought that the *Cello suites* (BWV 1007-1012) were designed to be that second collection.<sup>41</sup> Interestingly, Anna Magdalena's copy of the cello suites does not contain the numerical characteristics of a finally revised collection, nor does the numerical structure bear any resemblance to the *Six solos* for violin. In view of this lack of structural resemblance, and in the light of the parallel structure with the *Six solos*, I would like to propose that it is the *Six sonatas* (BWV 1014-1019) rather than the *Cello suites* that merit the title Libro Secondo.

<sup>40</sup> Note that the 2400 bars in the *Six solos* do not include repeats, whereas in the *Six sonatas* they do. This appears to be inconsistent, and I was unhappy with the result initially, but the parallel exists at more than one structural level and therefore must be taken seriously.

<sup>41</sup> On the basis of the title page of the copy Anna Magdalena Bach made for Schwanenberg (P268), on which she ascribes the subtitle 'Pars 1' to the *Six solos* and 'Pars 2' to the *Six cello suites*.

### v) Realisation and implications

Level 5	2400:2400	1:1
Level 4	1600:800 4:2	2:1
Level 3	408:816 and 272:408	1:2 and 2:3
Level 2	136:136 and 136:272	1:1 and 1:2

Example 13: The four levels of proportion found in *Six solos* (BWV 1001-1006)

The theory of proportional parallelism shows that Bach consciously manipulated the bar structure of his collections so that they are related to one another at different levels of their construction. For the *Six sonatas* (BWV 1014-1019) to have the same level 4 and 5 proportions as the *Six solos* (BWV 1001-1006), Bach would have had to make adjustments to only one of the works. We have no copies of a later version of the *Six solos*, whereas we have three different versions of the sixth (BWV 1019) of the *Six sonatas*. Although Bach first composed and performed the *Six sonatas* in Cöthen, he made a final revision of the collection at a later, unknown, date. The existence of earlier versions of the *Six sonatas* suggests that he may not originally have planned these two works to be companion pieces. The documented adjustments to the sixth sonata, however, were all that was required to manipulate the collection into a perfect companion for the *Six solos*.<sup>42</sup>

Since every example of proportion level 5 formed between two collections must have been made after the completion of the first of the two collections, it is not surprising to find that the second collection contains fewer levels of proportion than the first. The reason for this is obvious. The changes necessary to conform a collection to a specific bar total would destroy pre-formed proportions. The situation becomes more complex when the composer changes his numerical plan after parts of the collection have been published. There is evidence to suggest that on several occasions Bach did just this.<sup>43</sup>

The *Six solos* present a textbook case of four levels of proportion. Similar proportional correspondence at levels four and five, i.e. between collections and within a collection, appear in almost every work that Bach published or copied in fair hand, whereas they are rarely present in earlier versions. It seems therefore that the formation of proportions at these levels was part of Bach's revision procedure for a final polished version. Proportions at level two, formed between the movements of a work, on the other hand, are extremely common, regardless of whether the work is in compositional, revision or fair copy, and most remarkably in many of the church cantatas, including those surviving as

<sup>42</sup> Therefore, when Bach wrote Libro primo in 1720 on the score of the *Six solos*, he probably did not intend the *Six sonatas* to be Libro secondo. However, after he had made the revisions, whenever that was, I would not be surprised that he allocated the title Libro secondo to his fair copy of the *Six sonatas*.

<sup>43</sup> For example, the chronology of the engraving of the *Canonic Variations* BWV 769, as well as possibly *Clavierübung* I.



composing scores. This seems to indicate that creating overall balance and proportion through the bar structure was part of Bach's everyday technique.<sup>44</sup> Such simple addition of five or six terms would not have required a complex ground plan, and by the time the orchestral parts were to be transcribed, Bach and the copyists knew how many bars there were in each movement.

Level 5	Proportion formed by two collections
Level 4	Proportion formed in the collection as a whole
Level 3	Proportion formed between two works in a collection
Level 2	Proportion formed between movements of a work
Level 1	Proportion between sections of a movement

Example 14: The five different levels of proportions found in Bach's works

### vi) The theory of proportional parallelism

Proportional parallelism is a new concept in both theory and analysis and historical musicology.<sup>45</sup> The proportions I have discovered are parallel to each other in two dimensions of the work's construction. First, the all-encompassing proportions exist in time during a performance. That is, several levels of 1:1 and 1:2 proportions are enacted during a performance, although the proportions themselves cannot be measured by a literal unit of time. Secondly, they exist spatially. That is, the score itself stands as an architectural design with several levels of 1:1 and 1:2 proportions. I could have chosen the term 'simultaneous' rather than 'parallel' to qualify the nature of the proportions, but the implications of time in the word 'simultaneous' make it less apt.

Although the proportions co-exist in time and space, the question remains whether they can be experienced, and if they cannot be experienced, do they have any function at all? They are enacted in performance, but do they really create for the listener and performer a sense of 'good proportion in all parts'? Zedler's entry on beauty in architecture, published in Leipzig in 1743, refers the reader to Vitruvius and describes the principles of symmetry, of ordering of all parts to form and the use of the proportions of the human body, sounding very much like Mattheson's recommendations for the Capellmeister. The writings of Vitruvius (c.80–25 BCE) were frequently cited in Bach's time, and, as the article in Zedler shows, his principles for architecture were upheld as the standard of beauty. Alberti (1404–1472) was one among many who were strongly influenced by Vitruvius. It would be easy to assume that the spatial proportions recreated in

<sup>44</sup> It may also be a clue to another, currently unexplored, use of constructional proportions in Bach's work implied by the figures in Tables 2 and 3 from the early version of the *St Matthew Passion*, which are not present in Bach's final revised version of the work. I will be exploring these constructional proportions in the large vocal works from autumn 2007.

<sup>45</sup> The concept of parallelism would not have been foreign to composers in Bach's time. Zedler's dictionary lists a variety of terms that have 'parallel' at their root, including 'paralellistica methodus', 'loca parallela' and 'paralellismus' itself. The descriptions range from the use of parallelism in literature, hermeneutics, biblical interpretation and maths.

architecture in Bach's time were more tangible than those in music of the time, but if the great art historian Rudolf Wittkower is to be believed, this was not the case. Referring to Alberti's rather than Vitruvius' description of the proportions, he writes:

These proportions of one to two, two to three or three to four conform to the all-pervading law of harmony . . . It is obvious that such mathematical relations between plan and section cannot be correctly perceived when one walks about in a building. Alberti knew that, of course, quite as well as we do. We must therefore conclude that the harmonic perfection of the geometrical scheme represents an absolute value, independent of our subjective and transitory perception.<sup>46</sup>

Would Bach have agreed with Wittkower? Were the proportions in the *Six solos* designed to be perceived? If Wittkower is correct, the answer is 'no'. Just as the proportions in architecture between plan and section were not planned to be perceived, but to represent an absolute value, the musical proportions are a feature of the written score only.

### III Meaning

To create exact numerical proportions in a work or collection is not as complex or as obsessive as it may at first seem, although the revision procedure necessary to achieve the more ambitious levels of proportion would have taken forethought and some form of written numerical plan. What motivated Bach to introduce these parallel proportions? Was it a trick to stimulate his lively mind, well known for generating multiple permutations, while he had to endure the otherwise fairly mechanical and tedious process of revising a work? Or had he planned to make sets of perfect collections when he was a young man?

#### i) Perfection or beauty?

Perfection, or *Vollkommenheit*, was an important concept in Bach's time. Working from the biblical principle that God is the most perfect being, the concept of perfection was highly developed with widespread application, as is shown by its coverage over 150 columns in Zedler's dictionary.<sup>47</sup> The mathematical *numerus perfectus*, or *Vollkommene Zahl* (perfect number) is also discussed and, significantly, included in Walther's very small dictionary. Six is a perfect number.<sup>48</sup> Was Bach's occasional use of six works in a collection evidence of a conscious desire to create a perfect number, in an attempt to strive towards perfection? Intervals in music were expressed in terms of perfection: the perfect unison 1:1, the perfect octave

<sup>46</sup> Rudolf Wittkower, *Architectural Principles in the Age of Humanism* (Chicester: Academy Editions, 1988) 5<sup>th</sup> edn, p.18.

<sup>47</sup> See footnote 31 above and <http://www.zedler-lexikon.de/> 'Vollkommenheit', under which there are multiple sub-entries.

<sup>48</sup> In *The City of God*, Augustine observed: '6 is a number perfect in itself, and not because God created all things in six days; rather the inverse is true; God created all things in six days because the number is perfect. And it would remain perfect even if the work of the six days did not exist.' Simon Singh, *Fermat's Last Theorem* (London: Fourth Estate, 1998), p. 12.

1:2, and the perfect fifth 2:3. It is quite possible that Bach was seeking to attain perfection by creating perfect proportions in his collections at many structural levels.

The parallel proportions in the final revisions of Bach's collections may also have been his attempt to recreate 'beauty'. In the article 'Beauty in Architecture', the author writes:

Beauty in architecture is the perfection or a semblance of perfection that generates pleasure. . . . The Rules of beauty for a building are based on symmetry, or the proportional agreement and well-disposed ordering of all the parts of a building.<sup>49</sup>

a description very close to Mattheson's recommendation to the composer.<sup>50</sup> It is interesting to see that in Leipzig, 1742 beauty was defined in terms of perfection. Was motivated Bach to create perfection and beauty when he formed parallel proportions in his works?

### ii) Self-referentiality?

Besides the proportions being an indication of the composer striving to recreate perfection or beauty, it is also quite possible that Bach occasionally used features in the score to indicate his name. Composers and artists quite often placed a signature in their works. Sometimes the signature is clearly on the title pages,<sup>51</sup> and sometimes, as in the *Art of Fugue* (BWV 1080), through a disguised, yet recognisable, device. The family pun on the surname 'Bach' using the pitches b-a-c-h is recorded by Walther, whose informant was the Leipzig Bach.<sup>52</sup> We know that Sebastian used this in the unfinished movement BWV1080/20. Smend claimed to have found the name in many more compositions, not through the pitches b-a-c-h, but through the numerical equivalent of the name.<sup>53</sup> Unfortunately, Smend's idea encouraged the worst excess of unscientific musicology, in which any 41, 14 or 158 in Bach's works was seen to be Bach's signature. Examples 10 and 12 show that the keys for three consecutive works in the *Six solos* and the *Six sonatas* spell the name B-a-c: a pattern that is only visible once the collections have been reduced to level 5 proportion.<sup>54</sup>

<sup>49</sup> Zedler, *Universal Lexicon*, s.v. 'Schönheit in der Baukunst': 'Die Regeln der Schönheit eines Gebäudes kommen sonderlich auf die Symmetrie an, oder die proportionirliche Uebereinstimmung und wohlangebrachte Ordnung alle Theile eines Gebäudes'.

<sup>50</sup> See pp. 38 and 39 above.

<sup>51</sup> For example, it can be achieved through such a simple device as capitalisation: note the capitalisation in the title of this paper. It is crystal clear once you have noticed it, but not necessarily the first thing you would notice in an academic paper.

<sup>52</sup> According to the printed text of the *Musicalisches Lexicon* (Leipzig, 1732), that is. In Walther's own copy of his dictionary he made a few manuscript corrections, one of which was to state that it was the Jena Bach, i.e. Johann Nicolaus (1669–1753), and not the Leipzig Bach who told him about the b-a-c-h pun.

<sup>53</sup> 41= J. S. Bach, 14= Bach, 158=Johann Sebastian Bach.

<sup>54</sup> The term 'h' was not used universally in Bach's time. Some Germans did and others did not. See Joseph Riepel, *Anfangsgründe zur musicalischen Setzkunst* (1752, 1755) § *Grundregeln zur Tonordnung insgemein*.

The key pattern has also been noticed across the first two books of the *Clavierübung*, although in this case it spells b-a-c-h. The final 'h' is yet another

	<b>Bars (no repeats)</b>	<b>Key pattern</b>
Partita 1	249	B flat
Partita 2	378	c
Partita 3	342	a
Partita 4	422	
Partita 5	391	
Partita 6	396	
Italian Concerto	451	
French Overture	491 <b>3120 bars</b>	<b>h (b minor)</b>

Example 14: *Clavierübung* Parts 1 and 2, with 3120 bars and b-a-c-h key pattern

possible explanation for Bach's transposition of the French overture from c minor to b minor. The signature (b-a-c-h) unites the two collections and may contribute to the idea that Bach conceived of them as one large collection, and thus support the discovery of the level five 1:1 proportion with 3120:3120 bars uniting these two collections.

Letter-number equivalence was well-known and used in Bach's day in widely differing contexts, from school-boy arithmetic to the developed form of the poetical paragram. That the name b-a-c is formed by the first three letters of the alphabet would not have escaped the attention of any literate or numerate Bach family member. That Johann Sebastian was born on the 21<sup>st</sup> day of the third month (213) would also have been self-evident to the young intelligent schoolboy, as dates at the time were written in this order – 213 (bac and 21 March). Bach was a master of permutation. The coincidence of his birthdate and name, combined with the frequent recurrence of the numbers 1, 2 and 3 in the final totals of collections, forces me to ask whether Bach's use of the first three numbers in various permutations might occasionally represent his signature. The repetition of 3120 bars as the common total of his two important keyboard collections is striking. The further 'coincidence' that the *Six sonatas* for organ (BWV 525-530) has 1560 bars, exactly half of 3120, cannot be ignored.<sup>55</sup>

At least two levels of proportion exist in all Bach's published works, and at least two levels of proportion exist in the majority of the works that he transcribed into fair copy. Conversely, the majority of works surviving in autograph revision and compositional scores lack these two levels of proportion. Consequently, the theory of proportional parallelism becomes useful for musicology in at least six ways. The first is valid regardless of whether the proportions were designed by Bach or not.

<sup>55</sup> Bach transcribed the organ sonatas at the same time as he was composing and preparing the fifth and sixth partitas for publication, by which time he would have drawn up his final plan for the two keyboard collections.

1) The theory offers a new technique with which to demonstrate coherence and unity across a Bachian collection. The proportions can be appreciated both spatially and conceptually in time, explaining the unity within some of Bach's compositions that has frequently been sensed intuitively, but never scientifically demonstrated. The demonstration of unity is particularly attractive to the musical analyst, and therefore the theory could be classified as a 'historically-consistent analytical theory'.

The remaining uses are valid only if the proportions were designed by Bach.

2) The proportions can help demonstrate which scores Bach had finally revised into a state that he considered worthy of publication, although he might have left the score in a manuscript copy. The numerical structures of both *Das Wohltemperirte Klavier* (BWV 846-869) and *Aufrichtige Anleitung* (BWV 772-801) are clear and simple examples.<sup>56</sup> A more complex case, because of the source and reception history, is the 'Great 18' Chorale preludes for organ (BWV 651-668). The numerical proportions in the autograph score P271 demonstrate clearly that the first fifteen chorale preludes (BWV 651-665) have two levels of proportion with a structure of 1200 bars, showing that Bach had devised a collection of fifteen chorales, and not eighteen, as created by later copyists.<sup>57</sup> Conversely, the absence of 1:1 and 1:2 proportions in the structure of the harpsichord concertos BWV 1052-1058/9 P234, show that this was not a revised collection. This does not exclude the possibility that Bach had made a final revised collection; but if he did, it is lost.

3) The lack of a 1:1 or 1:2 proportion at two structural levels demonstrates that a score has not been revised. For example, the early versions of the Partitas BWV 827 and BWV 830 in Anna Magdalena's music book lack two levels of proportion and were adapted when they were grafted into the structure of the first part of *Clavierübung I* (BWV 825-830).

4) The principles of the theory can also be used to show whether a score in a copyist's hand was made from a finally revised Bach score or not. For example, Anna Magdalena's copy of the *Cello suites* (BWV 1007-1012) lacks proportions at levels three, four or five, showing that it is either a) not a faithful copy of Bach's score, or b) that Bach had not yet revised it to a state where he would consider it worthy of publication. The numerical structure of Altnickol's copy of the *Six sonatas* for harpsichord and violin (BWV 1014-1019), on the other hand, shows that this, and not the two earlier versions, was Bach's final revised version of this collection.<sup>58</sup>

5) The principles of the theory also explain the small but significant changes Bach made to the bar structure, as evidenced when comparing an early and later version of a score. Editors have frequently been at a loss to explain why Bach made these changes. Although we can assume that a musician of Bach's stature is unlikely to have made an 'unmusical' change, there are occasions when the alterations stand out. Hans Eppstein asked why Bach sets up a clearly logical

<sup>56</sup> See pp. 50-51 above

<sup>57</sup> As with all results, this is confirmed by the dating of Bach's entry of the chorales, and supported by evidence from the early publications.

<sup>58</sup> See p. 51 above.

pattern and then frequently breaks it.<sup>59</sup> The proportional levels provide an explanation.

6) Finally, the principles of the theory may provide an additional tool with which to piece together the compositional order of a collection, or the order of compilation of a collection. Several significant Bachian collections and compositions lack the proportional layers characteristic of his published works, which raises many questions about their status. It is particularly intriguing that the majority of the vocal works, the core of Bach's compositional duties as Thomaskantor, were not perfected.

The theory of proportional parallelism has nothing to do with symbolism. My research into the number alphabet, the golden section, and experiments with text and number were necessary avenues of inquiry in order to reach the main goal: a historically-plausible theory that might explain the unclear references to numerical processes in Bach's compositions. Having seen the levels of proportion in the *Six solos*, I now have to ask whether Bach's contemporaries knew all along that Bach ordered his compositions in this way. In a summary of how to compose a melody, Mattheson lists thirty-three guidelines, among which are the following: 'observe well the proportion of all parts, sections and divisions';<sup>60</sup> 'one should have the geometrical proportion of similar movements consciously before you, specifically the *numerus musicum*, i.e. retain exactly the measurement of the melody by numbers';<sup>61</sup> and 'the number of bars should be proportioned'.<sup>62</sup> In the context of the levels of proportions in the bar structure of the *Six solos*, these guidelines appear to have a specific meaning. Was Mattheson advocating proportional parallelism?

The discovery of Bach's use of parallel proportions gives Bach studies a new technique with which to demonstrate unity within his works. The usefulness of proportional parallelism to the editor and interpreter of Bach's music is limited, however, unless one fundamental question can be answered: is proportional parallelism analytical coincidence or Bach's design? It is every theorist's dream to be able to conclude a project with the three letters Q.E.D. Unfortunately, in the absence of a document in Bach's hand describing the proportions, this dream remains elusive. I must therefore call on the tried and tested legal model, and invite the reader to weigh up the evidence for and against.<sup>63</sup>

<sup>59</sup> Hans Eppstein, 'Fragen der Ordnungsprinzipien in Bachs Köthener und Leipziger Instrumentalsammlungen', *Leipziger Beiträge zur Bach Forschung. Konferenz Bericht*, ed. Ulrich Leisinger (Hildesheim; 2002), 131–5. On p. 133 he writes about the sixth sonata BWV 1019, asking 'Was also mag die Ursache für Bachs Vorgehen sein?' The proportional layers provide an explanation.

<sup>60</sup> Mattheson, *Capellmeister* (1739), p. 141, §52, 5. 'Den Verhalt aller Theile, Glieder und Gliedmassen wol beobachten'.

<sup>61</sup> *ibid.* p. 141, §51, 2. 'Auch den geometrischen Verhalt gewisser ähnlicher Sätze, nemlich den *numerus musicum*, d.i. die melodische Zahl-Masse genau beibehalten'.

<sup>62</sup> *ibid.* p. 141, §50, 4 'Soll der Tacte Anzahl einen Verhalt haben'.

<sup>63</sup> Tempting as it is to give numerical results for all Bach's works, it is not possible to do so with any integrity without also including a wealth of source material. I hope that the small sample of results and sources given in this paper will nevertheless be sufficient to generate valuable discussion until *Bach's Numbers Explained* (in preparation) is published.